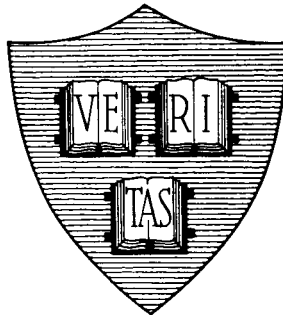


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Division of Engineering and Applied Physics
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March 1967

Division of Engineering and Applied Physics

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Nonr-1866(16)	The Steering Committee
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Related Contracts

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ARPA SD-88	H. Brooks
DA-ARO-D-31-124-G697	N. Bloembergen
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Nonr-1866(10)	H. Brooks and W. Paul
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I. VERY LOW FREQUENCY PROPAGATION

J. A. Pierce

Contract Nonr-1866(07)

1. Navigation

Measurement of the phase of a modulation frequency was mentioned in the last report. In navigation, this technique may be useful either in itself or for identification of one among several cycles of a higher frequency.

A suitable signal at $226 \frac{2}{3}$ c/s has been transmitted from Trinidad for several months. This signal appears as phase modulation of a carrier at 13.6 kc/s. The modulation index is extremely low, about 0.15, because of the high Q of the tuning system in the transmitting system. There are, therefore, only a few watts in the transmitted side-bands, and our first problem is to extract the desired signal from the high-noise environment.

This is done by using a broad-band limiter to reduce the impact of high noise peaks, and by having a limiter-discriminator-limiter conformation in the receiver. This latter is necessary because the "instantaneous frequency" of the signal varies through only a few tens of cycles while the receiver bandwidth must be several hundred cycles to pass the required sidebands. The second limiter reduces the wideband noise impulses to the size of the demodulated signal, before filtering, and greatly reduces the requirement for dynamic range in the phase-measuring equipment.

The combination appears to be satisfactory, although with only a single transmission path it is impossible to be sure that all observed phase changes are caused by propagation rather than by phase drifts and effects produced by noise saturation in the receiver.

It may be said at once that the $226 \frac{2}{3}$ c/s modulation is entirely satisfactory for the primary purpose of identifying the cycle of the Omega frequency at $1133 \frac{1}{3}$ c/s. The largest variations that have ever been observed are of the order of 200 microseconds in time, or less than $\frac{1}{4}$ of the period of $1133 \frac{1}{3}$ c/s. It is therefore clear that an essentially instantaneous measurement of the modulation phase is adequate for lane identification.

It is, unfortunately, much less likely that the accuracy with which $226 \frac{2}{3}$ c/s can be measured is enough to make its direct use for navigation of much interest. The reasons for this conclusion are shown in three figures. Fig. 1 shows the diurnal average and standard deviation for a series of measurements over eight days in July. We need not here pay much attention to the shape of the average curve, although the indications of beats between two (or more) modes of propagation are of some interest. The standard deviation is seen to be about 20 μ s for the entire day except for a pre-sunrise period. This level (20 μ s) corresponds to root mean-square positional errors of about three miles in a navigation system, - errors that would permit the system to be useful for many purposes.

A longer run, made soon after that of Fig. 1, however, gives quite different results, as shown in Fig. 2. Here there is no great change in the mean curve (although the beats are greatly reduced in importance) but the standard deviation is near 50 μ s (or 8 miles in position). This

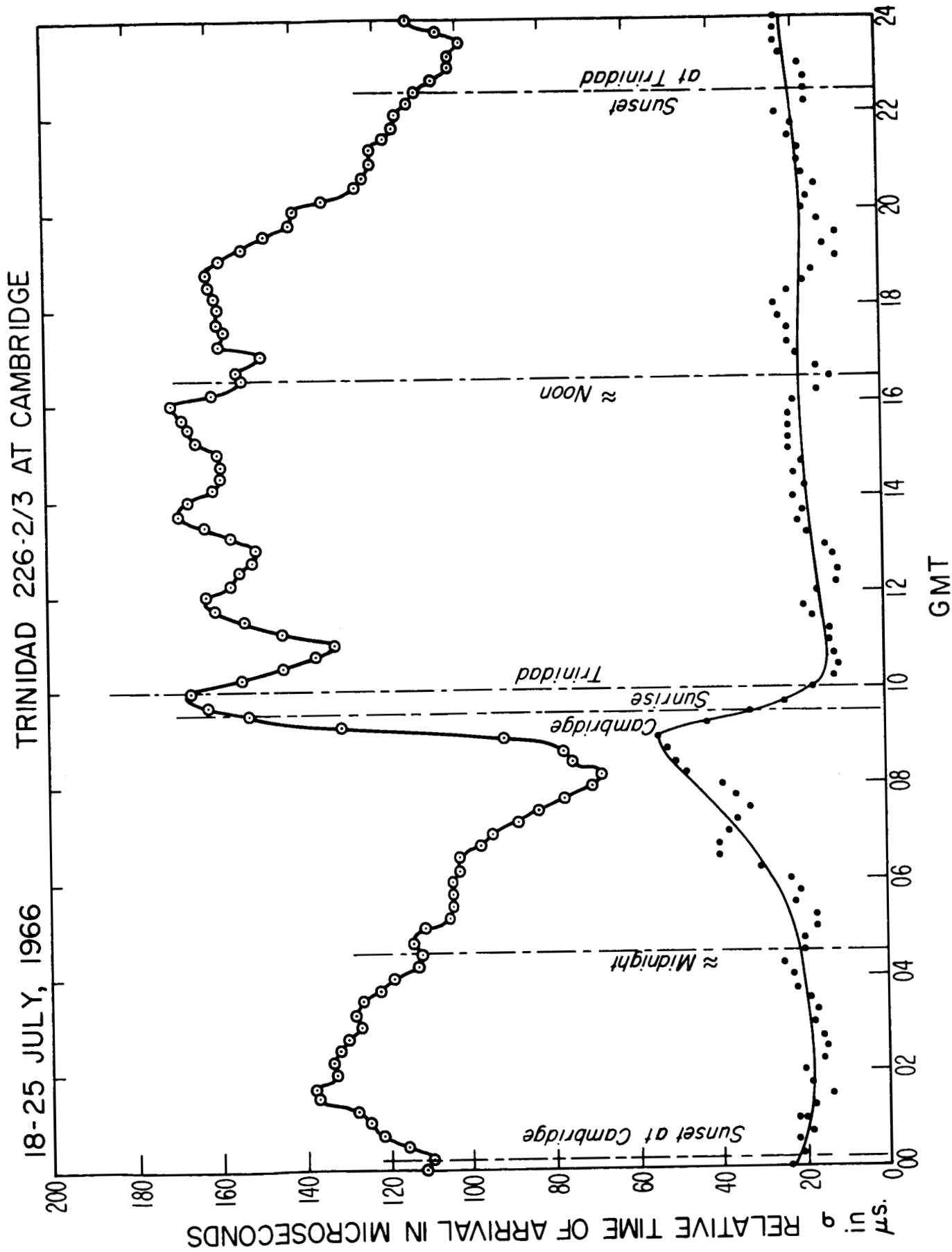


FIGURE 1

TRINIDAD 226-2/3 AT CAMBRIDGE

30 July-18 August 1966

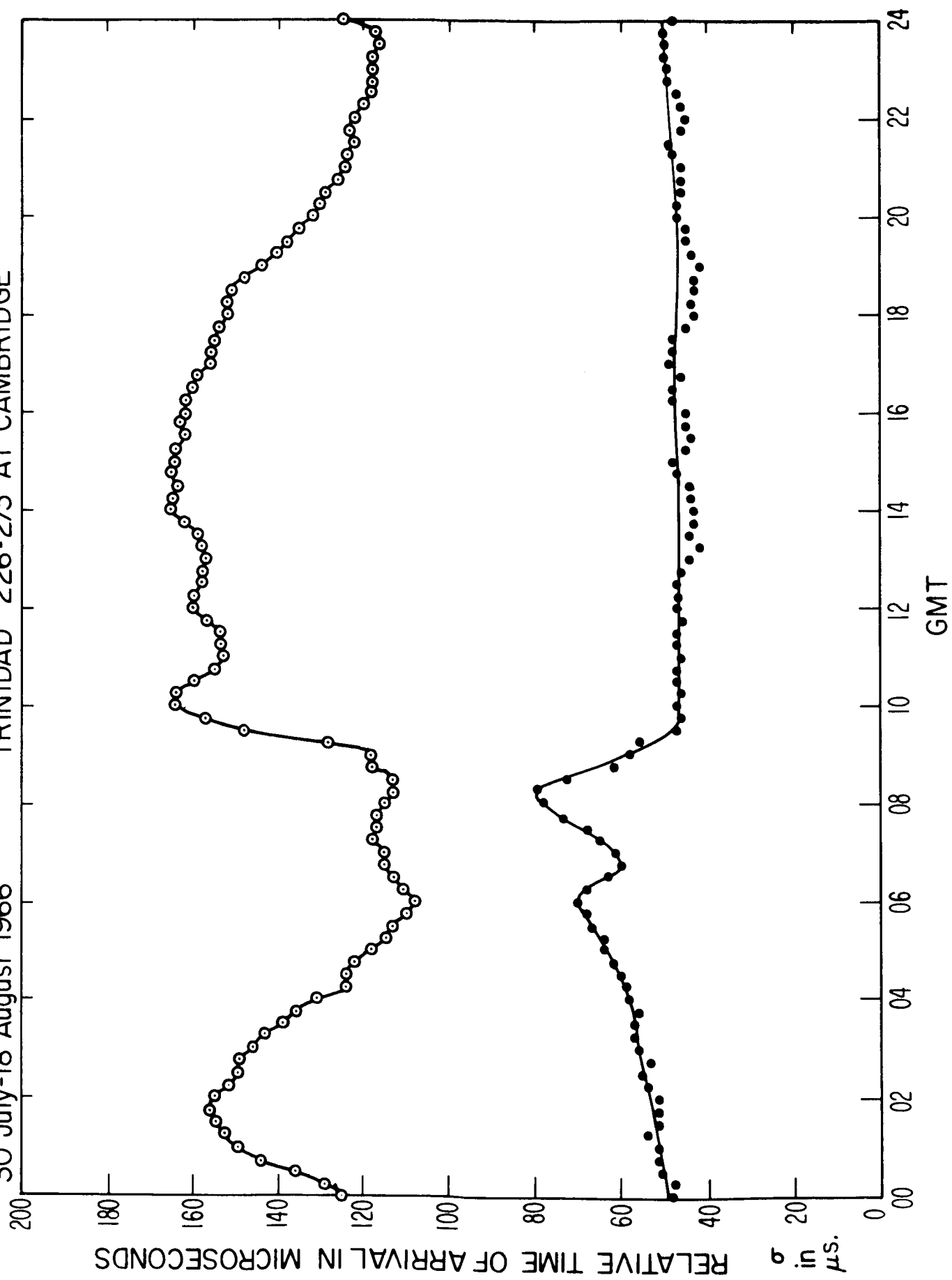


FIGURE 2

large level and the general uniformity throughout the day suggest that the primary variation is not, in fact, diurnal. To check this hypothesis the data (corrected for the mean diurnal variation) were plotted as a function of data in Fig. 3. It is seen in this figure that the received phase wanders relatively slowly throughout a range of ± 100 microseconds. The quasi-period of several days seems to suggest that the cause may be variations in magnetic activity. Such a mechanism might well act by changing the ratio of first to second mode of propagation. If so, the effect might well be smaller at larger distances, but it would be too much to expect that the variations would actually decrease with increasing distance.

It is, as suggested above, possible that part of these deviations may be caused by varying phase delay in the receiving system. It is not likely, if so, that greatly improved results could be obtained without a considerably more complex and expensive receiver. We must, therefore, reluctantly conclude that use of the relatively simple phase modulation alone for navigation is not likely to give results of satisfactory accuracy. It will, therefore, be necessary to restrict the use of such a frequency to lane identification (for which it is entirely adequate) and to use a somewhat more complex receiver for accurate position fixing.

2. Regulation of Oscillator Frequency

The excellent results of the absolute phase system of Omega that were described in Report No. 72 have not been maintained. It is clear that there is nothing wrong with the theory of absolute phase, but its implementation has met with several difficulties. These are:

- a. Inadequate sampling chiefly the fact that all stations do not

TRINIDAD 226-2/3 c/s MODULATION PHASE AT CAMBRIDGE

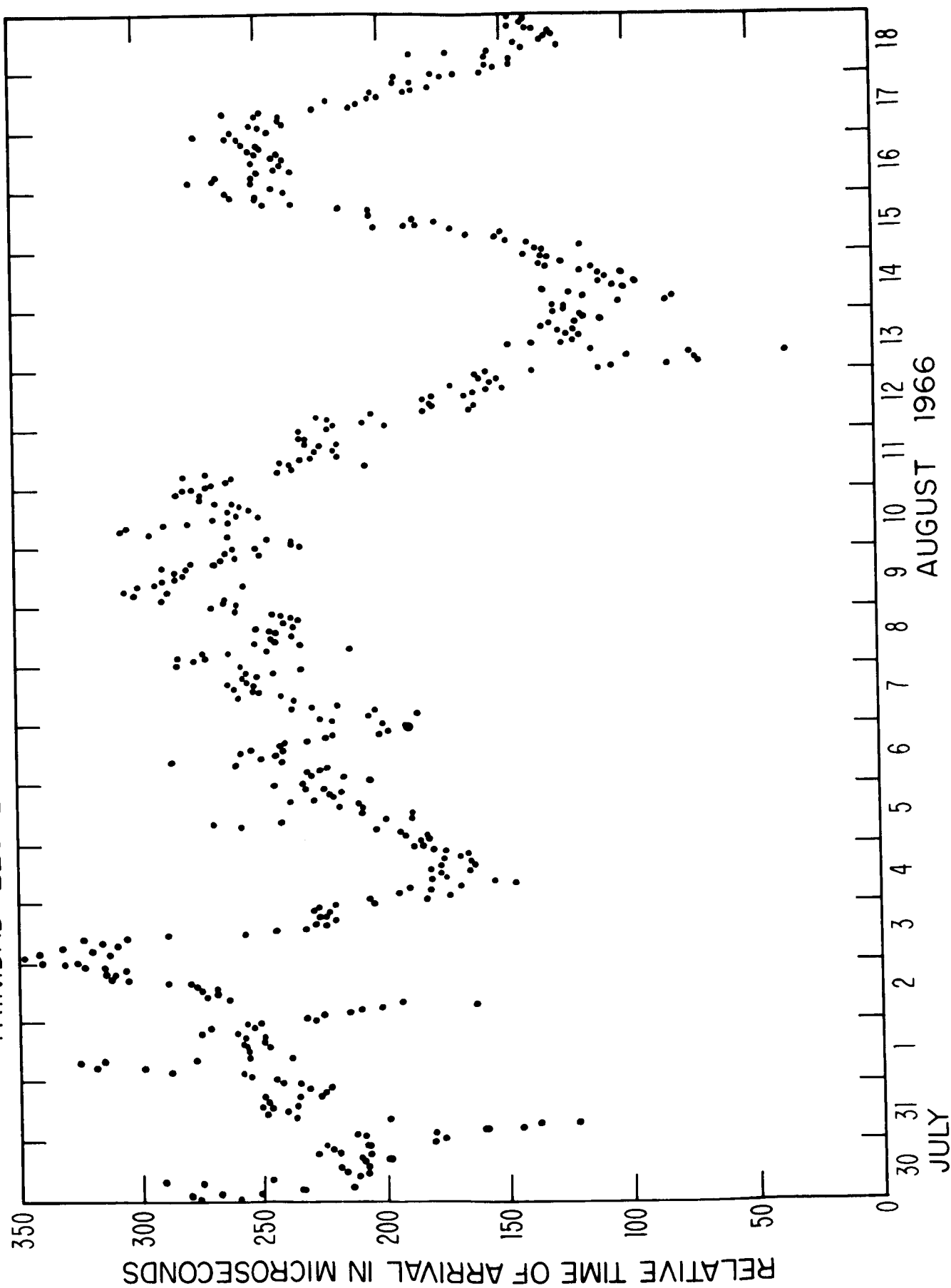


FIGURE 3

continuously examine the phase of all signals from all other stations. The data used are therefore not homogeneous and exceptional transmission conditions on one or more of the paths can lead to severe transients.

b. The predictions with which observations are compared have turned out to be less accurate in the fall than in the spring.

c. The time of arrival of signals from Norway, that traverse much of the auroral zone, have been much less stable than we had hoped.

d. The arithmetic base on which the station phases have been steered, has been shown to have much too short a time constant. In other words, we should put much more reliance upon the inherent stability of the frequency sources and allow exceptional events in propagation to have much less influence. A number of ways of modifying the arithmetic are under examination. This problem needs careful study, because inter-communication between stations is somewhat slow, and it is essential that the station crews make no errors in applying the technique. In fact, the limiting factor is probably human impatience. When working with a time constant of a month or so, an operator is all too likely to think that he can achieve a correct result more quickly by over-riding the mechanism. Such action may be dangerous when, as in this case, the noise is large and the signal small.

In spite of all these difficulties, the Omega synchronism has been maintained for about a year, with most deviations not larger than ten microseconds. This is, in itself, creditable, but with propagational vagaries having standard deviations less than ten microseconds it becomes important to keep the errors of synchronism well below that value.

3. Transmission Time

It has gradually become clear that our earlier measurements of the time of arrival of a difference frequency (specifically, the differences between 10200 c/s and 13600 c/s) have included too many instrumental errors. Most of these have come from the fact that the two carrier frequencies were recorded independently before the difference between them was taken. Thus, minor errors in the calibration of the recording devices or errors in recording the time of the independent observations might have a large effect upon the deduced phase of the difference frequency. This is particularly true if the correlation coefficient between the transmission times of the two carriers is positive and large, as it seems that it may be.

We have, therefore, made a new receiver that takes the differences instantaneously and independently records the phases of the two carriers and of the difference frequency. A separate section of the receiver tracks each of the four present Omega stations, and the primary differences between stations are also recorded. There are thus 21 outputs; 3 frequencies for each of four stations, and 3 frequencies for each of three differences between stations. It seems probable to us that any propagational phenomenon can thus be attributed to its proper source.

The analog receiver is much less complex than it sounds. There are two receiving channels (one for each carrier frequency) and two servos each of which is time-shared between the four stations.

At the end of this report period the equipment is beginning to operate. Although no data have yet been recorded, manual observations suggest that we may soon have good news to report.

II. ELECTRON AND SOLID STATE PHYSICS

Personnel

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Assoc. Prof. P. S. Pershan	Mr. P. Lallemant
Dr. G. D. Boyd	Mr. C. H. Lee
Dr. G. E. Durand	Mr. L. Malmstrom
Dr. A. S. Pine	Mr. J. Merz
Mr. G. Bret	Mr. A. Oseroff
Mr. R. Callender	Mr. H. S. Simon
Mr. P. Eisenberger	Mrs. P. Simova
Mr. W. N. Furey, Jr.	Mr. J. J. Wynne

1. Second Harmonic Reflected Light. H. S. Simon,
Contract Nonr-1866(16).

Second Harmonic Generation (SHG) of light by a Q-switched laser beam from the surface of various metals has been investigated. SHG has been observed from liquid Hg, single crystal Sb, and single crystal Bi. The magnitude of these signals is a factor of 5-10 below that from Ag^{*}, and represents approximately 4-8 second harmonic photons for each laser pulse. SHG has not yet been observed from ferromagnetic metals such as Ni and Fe. Preparation of a Na sample with a highly reflecting surface is being carried out.

2. Nonlinear Electroreflectance in Silicon and Silver. C. H. Lee
and N. Bloembergen, Contract Nonr 1866(16).

It has been observed that the intensity of Second Harmonic Light (SHI) produced in reflection by a Q-switched ruby laser beam incident

* R. K. Chang, N. Bloembergen, and C. H. Lee
Phys. Rev. Letters 16, 986(1966)

on silver and silicon surfaces shows a significant variation when a dc electric field is applied normal to the surface.¹ For both p-type and n-type silicon the SHI increases quadratically with E_{dc} . The minimum is near zero applied voltage. An enhancement of a factor of eight in SHI has been observed. The experiments were also carried out at 8658 \AA , by using NaOH, HCl instead of 0.1 N KCl as electrolytic solution, by changing different crystallographic orientations, and by using different polarizations of both the incident fundamental and the second harmonic light. In silver the effect interferes with the surface electric dipole term and shows dispersive characteristics.

The effect is caused by a second harmonic polarization $P(2\omega) = \chi E_{dc} E_L E_L$, where E_{dc} is the self-consistent screened dc electric field in the surface layer. This nonlinear electro-reflectance effect may provide a new tool for the investigation of surface phenomena since it depends on the convolution of laser field and dc electric field within the optical absorption depth of light.

The second harmonic generation of light from the surface of metals always has a low intensity. By utilizing multiple reflection we have seen a factor of six increase in SHI with only three reflections. The phase matching of fundamental and second harmonic on successive reflections is accomplished by varying the air pressure between the 2 parallel silver mirrors. This multiple reflection technique is a powerful means to improve the signal to noise ratio. Therefore it may be useful for the detection of SHG from metals such as nickel and iron.

¹A communication on nonlinear electroreflectance has been submitted for publication in Physical Review Letters.

3. Optical Properties of Liquid Crystals. G. Durand and C. H. Lee, Contract Nonr-1866(16).

The aim of the work is the investigation of the optical properties of liquid crystals, using laser sources. Light scattering experiments have given in the past some information on the structure of these mesomorphic states and the new characteristics of laser light should help in making some progress in this domain.

We have developed techniques to prepare samples of various nematic and cholesteric liquid crystals. These samples have been oriented using the surface technique of Chatelain. According to "the swarm" theory of Ornstein and Kast, the nematic mesophase is caused by cooperative effects between the static electric dipoles of the molecules. An investigation of the second harmonic generation by a Q-switch ruby laser on a surface and D. C. electric field oriented sample of azoxydianisole failed to see this correlation. The level of the signal, if any, is not significantly larger than what could be expected from any other polar liquid. Similar experiments are planned for cholesteric liquids. The ordering near the phase transition temperature will be studied by Rayleigh scattered light from a gas laser.

4. Stimulated Raman Effect. N. Bloembergen, G. Bret^{*}, P. Lallemand, and P. Simova[†], Contract Nonr 1866(28) and ARPA SD-88.

A summary of the work done previously on the stimulated raman

^{*†} Mr. Bret, on leave from the Compagnie de Telegraphic Sans Fils participated in this project in September and October 1966. Mrs. Simova is an exchange visitor from the Bulgarian Academy of Sciences and has participated in this project from October 1966.

effect in optically anisotropic liquids was submitted as a Ph. D. thesis which was defended on November 5, 1966 at the University of Paris by P. Lallemand.

A new laser system from Maser Optics has been installed and tested. Preliminary tests involved the use of a rotating mirror Q-switching device, but damages occurred to the ruby rod. A probable cause may have been the high reflectivity of the laser cavity. We now use a cryptocyanine chemical Q-switch that gives satisfactory results.

High pressure cells have been built in order to study the stimulated raman effect in hydrogen gas. We have performed two kinds of experiments.

1. Measurement of the raman frequency of the $Q_{01}(1)$ line as a function of pressure and temperature.
2. Measurement of gain in an amplifier as a function of length, pressure and frequency.

The frequency of the Stokes light generated in a cell of variable pressure and temperature is measured with a Fabry-Perot interferometer, using as a reference, the raman light emitted by another cell at constant pressure and temperature. The results are in good agreement with those obtained by spontaneous raman scattering.

The raman amplifier is operated in two different geometries where the direction of propagation of the laser and stokes beams are respectively the same or opposite. Most of the work has been done on the backward wave amplifier. We have measured the gain as a function of frequency by using different pressures in the oscillator and

the amplifier. This provides a measurement of the line width.

The results are described in a letter:

Pressure induced line shift and collisional narrowing in hydrogen gas determined by stimulated raman emission, P. Lallemand, P. Simova, G. Bret, Phys. Rev. Letters 17, p. 1239-1241 (1966).

The gain as a function of amplifier's length and pressure has been measured and is found to be in fairly good agreement with theory, without need to consider distortions of the laser beam. Results and experimental details are described in the first part of a paper submitted to the IEEE Journal of Quantum Electronics, "Controlled Stimulated Raman Amplification and Oscillation in Hydrogen gas." N. Bloembergen, G. Bret, P. Lallemand, A. Pine, P. Simova.

5. Nonlinear Optical Investigations with a CO₂ Laser.

J. J. Wynne and G. D. Boyd, Contract Nonr-1866(16) and Nonr-1866(28).

A high power continuous and/or pulsed CO₂ laser has been designed and constructed and is being used to study various non-linear optical phenomena. The laser output at 10.6 microns in the infrared may be used to study in transmission, nonlinearities in many materials not transparent in the visible.

Second Harmonic Generation in GaAs has been observed. Under uniaxial compression along a cubic axis GaAs becomes uniaxial and displays birefringence. The induced birefringence has been shown to be negative so that the phase matching condition between an ordinary fundamental and extraordinary second harmonic is approached. The magnitude of this effect has been roughly determined and phase matching

is not possible due to fracturing of the crystal above approximately 5000 bars of pressure.

This same study will be undertaken on InAs, CdS, CdSe and possibly Te and ZnTe. For these crystals as well as for GaAs accurate quantitative measures of this elasto-optical effect will be made.

For the future we will try to observe spontaneous Raman scattering from lattice vibrations using the 10.6 micron (944 cm^{-1}) laser. Totally symmetric vibrations with covalent bonding are expected to have the largest polarizability and thus Ge, Si and perhaps Te are the most promising candidates. Ge has a vibration frequency of 301 cm^{-1} . Intensities of 10 megawatt/cm^2 can be obtained at the focus of a Q-switched CO_2 laser. This leads to the possibility of observing small forward Stokes gain. The availability of excellent high quality Ge suggests low loss resonators and consequently stimulated Raman laser action.

6. Stimulated Brillouin and Raman Effects. A. S. Pine, Contracts Nonr 1866(28) and ARPA SD-88.

In the past six months two projects in nonlinear optics have been completed and two new projects have been started. Early in this period, work was concluded on optical heterodyne detection of stimulated Brillouin scattering in quartz, the results of which have been published (IEEE Jour. of Quant. Elect., QE-2, 673 (1966)). In this project feasibility was demonstrated for a sensitive and very precise means to study the high frequency acoustic spectrum of materials. Additionally

the optical beat employed in this experiment extends the frequency range for heterodyning to the K-band microwave region.

The second experiment was a study of the stimulated raman effect in hydrogen gas under controlled conditions. Oscillation was achieved in a resonator placed transverse to a laser beam. Various molecular excitations could be studied using selective mirror reflectivity and orientation. Contrary to many previous reports of anomalies in hydrogen in a longitudinal oscillator, the results of this experiment are in good agreement with theory. A joint report with Bloembergen, Bret, Lallemand, and Simova has been submitted for publication in the IEEE Journal of Quantum Electronics.

An experiment to measure spin-lattice interactions using the Brillouin effect has begun. The interactions of paramagnetic ions with phonons is manifest in the absorption and dispersion of sound. These quantities are detectible using light scattering techniques. In particular the magnetic field dependence of the phase velocity of sound can be obtained from stimulated Brillouin frequency shifts. Failure of the cryogenic system and a lack of suitable crystals have so far prevented progress on this project: work is continuing to correct these difficulties.

A joint project with Dr. George Durand is being conducted on Brillouin scattering from thermal phonons in solids. The purpose of this work is to obtain acoustic relaxation data at hypersonic frequencies at elevated temperatures. Strong phonon absorption prevents direct measurement by the usual ultrasonic techniques. Special low-noise photomultipliers and high resolution interferometers (compared to similar measurements on liquids already made) are necessary because of the small Brillouin cross section and narrow linewidth in solids.

7. Optically Induced Magnetizations in Paramagnetic Crystals.

L. Malmstrom, Contract Nonr -1866(16).

Crystals have been prepared, and equipment set up, for measuring the Faraday and Voigt effects in a number of rare earth doped calcium fluorides. On the basis of these results we hope to suggest other experiments in support of the ideas outlined in the previous reports.

8. Optical Spectroscopy of Divalent Rare Earths in Ionic Crystals.

J. Merz, Contract Nonr-1866(16).

Optical absorption and thermoluminescence measurements are used to study the X-ray induced reduction of trivalent rare-earth ions to the divalent state in CaF_2 . Thermoluminescent glow curves for the thirteen available lanthanide ions are measured between 80° and 450°K . These are found to be remarkably similar, with glow peaks occurring at nearly the same temperatures for each of the ions. The activation energies for each of the glow peaks is estimated. High resolution measurements of the spectra of each glow peak show that the emission is identical to the fluorescence of trivalent rare-earth ions, and determine the site symmetry of the emitting ion. The results of these spectral measurements can be interpreted in a consistent manner. All of the glow emission below room temperature originates from trivalent rare-earth ions in cubic sites, while the glow spectra above room temperature indicates tetragonal symmetry. Vibrational side bands are observed for some of the ions. In the model proposed to explain this data irradiation at 77°K produces divalent rare-earth ions and hole centers. Heating allows a hole to diffuse to the site of a divalent

rare earth. The hole and extra electron recombine, leaving an excited trivalent rare-earth ion. Decay of this ion to its ground state results in the observed thermoluminescence. For the low temperature peaks, the hole center must leave the rare-earth in a cubic site. For example, a V_K center would have this property, and the evidence suggests that similar holes trapped at lattice sites are associated with the glow peaks below room temperature. At high temperatures, the tetragonal spectra could result from the diffusion of F^0 atoms. Optical absorption bands are found for both the divalent rare earths and the hole centers, and these bands bleach as the crystal is heated. Above 350°K, several of the rare earths exhibit anomalous absorption effects which result from more complicated configurations of holes. Thermoluminescence studies of the isomorphs SrF_2 and BaF_2 are consistent with the model proposed for CaF_2 .

A preliminary report of this work has been given at the Conference on the Optical Properties of Ionic Crystals at Johns Hopkins in September of 1966. That report should be published in the Proceedings of the Conference. A manuscript for submission to the technical journals is now in preparation.

9. Semiconducting Properties of Rare Earth Doped CdF_2 .

P. Eisenberger, Contract Nonr-1866(16).

When single crystals of CdF_2 , doped with rare earth ions, are baked in a cadmium atmosphere, the material becomes semiconducting. At low temperature, the conduction is frozen out and the conduction electrons are presumably trapped at various sites. Electron

spin resonance studies of Gadolinium doped CdF_2 reveal positive evidence that the electrons are attracted to the divalent rare earths by electrostatic effects but that they remain localized on cadmium ions. Further optical and photoconductivity data have also been obtained for this material.

10. Magneto-optical Effects in Solids. F. Kahn, Contract Nonr-1866(16).

The design and construction of the instrument for measuring magneto-optical effects in reflection is continuing. This equipment will be evaluated by measuring ferromagnetic and antiferromagnetic insulators which are known to have important electronic transitions in the optical range. Examples of this are the various garnets and iron or chromium oxides.

11. Raman Spectroscopy from Mixed Crystals. B. Lacina, Contract ARPA SD-88.

The Raman spectra of mixed crystals of CaF_2 - SrF_2 and SrF_2 - BaF_2 have been studied. In contrast to the vibrational spectrum of other mixed crystals the frequency of Raman active mode appears to go linearly with concentration. Our experimental configuration is being improved so that line width studies can be made at low temperatures. A preliminary account of this work has been published. Phys. Rev. Letters 17, p. 755 (1966).

12. Raman Spectroscopy in Magnetic Crystals. A. Oseroff, Contract ARPA SD-88.

The Argon laser, designed and constructed by Dr. G. A. Brooker, has been operated successfully. The SPEX double monochromator purchased for Raman spectroscopy has arrived and preliminary tests show it to be operating satisfactorily. We hope soon to begin Raman studies on various magnetic crystals.

13. Electronic Effects in Transition Metal Oxides. R. Callender, Contract Nonr-1866(16).

The majority of effort on this project has been concerned with the selection of suitable materials. We have now obtained some crystals of tin oxide with reasonably high electron concentration. Preliminary experiments indicated the samples on hand were too highly degenerate to be of any use and we have now obtained crystals with lower free carrier concentrations.

14. Band Theory of Ferromagnetic Anisotropy in Nickel.
W. N. Furey, Jr. Contract Nonr-1866(16) and ARPA SD-88.

The anisotropy constants K_1 and K_2 have been calculated by including spin-orbit coupling in an interpolation scheme¹ for the band structure of ferromagnetic nickel. This numerical calculation extends

¹ L. Hodges, H. Ehrenreich, and N. D. Lang, Phys. Rev. 152, 505 (1966).

an earlier theory² by using improved bands and by considering the effects of spin-orbit coupling on the occupation of states near the Fermi surface.³ The calculated value of K_1 at $T = 0$ agrees well with experiment. It was found that the one-electron states in a limited region of the Brillouin zone near the point X account for practically all of the anisotropic part of the crystal energy. In this region the bands are degenerate, or nearly so. Because of degeneracy, it is found that the anisotropy constants can exhibit strong temperature dependence. The calculation also gives $K_1 \approx K_2$, a result which is supported experimentally, but which is hard to understand on the basis of earlier models which used non-degenerate perturbation theory.

² G. C. Fletcher, Proc. Phys. Soc. (London) A67, 505 (1954).

³ J. C. Slonczewski, J. Phys. Soc. Japan 17, Suppl. B-1, 34 (1962).

III. AUTOMATIC CONTROL

Personnel

Prof. A. E. Bryson, Jr.	Mr. J. L. Henrikson
Prof. R. E. Kronauer	Mr. J. S. Lee
Assoc. Prof. Y. C. Ho	Mr. R. J. McLaughlin
Asst. Prof. A. R. Dobell	Mr. R. K. Mehra
Dr. K. J. Bullock	Mr. C. P. Neuman
Mr. R. Behn	Mr. P. M. Newbold
Mr. S. Benton	Mr. J. Speyer
Mr. C. C. Blaydon	Mr. R. Subramanian
Mr. J. Budelis	Mr. J. Turnbull
Mr. R. B. Cambell	Mr. L. K. Williams
Mr. M. Desai	Mr. D. H. Winfield

III. A. Systems Analysis and Control

1. Adaptive Systems R. E. Kronauer, L. K. Williams, and K. I. Bullock
Contract Nonr-1866(16).

Analog computer studies have been performed on parameter-perturbation adaptive techniques where the basic plant was third and fifth order. The design criteria for the adaptive loop proposed by Kronauer and Drew* were found to yield the best adaptation. In cases where the regime of satisfactory plant operation was bounded by an instability it was possible to start with the plant in the unstable regime and adapt satisfactorily with a probability which decreased as the degree of initial instability was increased.

Work is in progress which is aimed at delineating accurately the advantages which accrue to adaptive schemes which depend on multiple replicas of the basic plant. In particular, what is the incremental advantage for each additional replica ?

* "Design of the Adaptive Feedback Loop in Parameter-Perturbation Adaptive Controls," R. E. Kronauer and P. G. Drew, IFAC Symposium, 1965.

2. Nonlinear Oscillations. R. W. Kronauer, R. J. McLaughlin and R. Subramanian. Contract Nonr-1866(16) and NsG-559 .

A class of nonlinear dynamical problems arising in the theory of dynamic buckling is being studied. These systems exhibit oscillatory motions for small amplitudes but divergent behavior in one or more of the coordinates at large amplitudes. The aim is to bound the initial conditions which lead to divergence. Even the simplest realization of this kind of system is fourth order and has four nondimensional parameters. The analog computer is being used extensively. Among the interesting results to date is the discovery of certain "trapped" modes where extraordinarily large initial energies fail to produce divergence.

3. Optical Storage and Processing . S. Benton and R. E. Kronauer, Contract Nonr-1866(16). (Partially supported by IBM).

The predictions of the grain noise analysis for uniformly exposed emulsions have been confirmed experimentally. The dependence of total grain noise on the average amplitude transmittance displays the predicted maximum near the 50% level, and for the emulsions tested the grains exhibit an opacity near unity. The asymptotic high wave-number spectrum shows the grains to be of the sharp-edge variety.

The analysis for signals varying slowly (compared to the grain dimension) is complete. The analysis for signals comparable to the grain dimension is being pursued.

III. B. Automatic Control

1. Numerical Solution of Optimal Control Problems. R. K. Mehra, M. N. Desai, and A. E. Bryson, Jr., Contract Nonr-1866(16).

Two computer programs were developed for numerical solution of nonlinear programming problems with constraints, one using Davidon's variable metric approach and the other using the conjugate gradient approach. Several specific problems were solved to check out the program and to test the relative efficiency of the two approaches.

Another general computer program was developed for numerical solution of optimal programming problems (calculus of variations problems) with terminal constraints using a conjugate gradient method similar to Lasdon's. Several specific problems were solved, showing improved efficiency over the ordinary gradient method.

As an example of quadratic synthesis, the control logic for a lateral motion autopilot was designed for steady flight of an airplane disturbed by random cross-winds. The RIAS-ASP program was used to solve the fifth order Riccati equation in order to obtain the optimal feedback gains. Using these gains, a statistical analysis for the mean square value of the state and control variables was carried out. Parameters in the quadratic criterion function were chosen in order to get reasonable answers for the mean square values of state and control variables.

References

1. W. C. Davidon, "Variable Metric Method for Minimization," AEC Research and Development Report ANL-5990, 1959.
2. L. S. Lasdon, S. K. Mitter, and A. D. Warren, "The Conjugate Gradient Method for Optimal Control Problems," to be published.

2. Comparison of Quadratic Synthesis with Pole-Zero Synthesis .

J. Turnbull and A. E. Bryson, Jr., Contract Nonr-1866(16).

For some years now tracking radars designed according to conventional techniques have been operating quite successfully. A question of interest, however, is whether the techniques of the so-called modern or time-domain control theory can be used to pick feedback gains that will result in a system of higher precision.

The approach has been as follows: A particular linear model has been decided on for the antenna drive and the measurements which include additive noise. A stochastic model for the target has been developed. A quadratic performance index has been chosen to penalize both tracking error and power input to the antenna drive. Feedback gains are then chosen so as to minimize the expected value of this performance index.

In the future, the RIAS-ASP program will be used to obtain numerical estimates of the system's performance which can be compared with the performance figures for conventionally designed tracking radars.

3. Optimal Economic Growth Plans . J. Budelis, A. R. Dobell, and A. E. Bryson, Jr., Contract Nonr-1866(16).

Until recently economists working on optimal growth planning have not taken advantage of progress in optimal control theory. During the last six months a report has been written which summarizes some of the past work on growth planning from the point of view of optimal control theory. The following articles have been analyzed:

F. P. Ramsey, "A Mathematical Theory of Saving," *Economic Journal* Vol. 38, 1928, pp. 543-559, Tjalling C. Koopmans, "On the Concept of Optimal Economic Growth," Cowles Foundation Discussion Paper No. 163, December 6, 1963, Appendix No. 163-A, December 19, 1963; David Cess, "Optimum Economic Growth in an Aggregative Model of Capital Accumulation: A Turnpike Theorem," Cowles Foundation Discussion Paper No. 178, November 9, 1964.

With this as background, extensions to more realistic (and hence more difficult) economic models will be attempted.

4. Numerical Solution of Nonlinear Programming Problems. D. H. Winfield, Contract Nonr-1866(16) (supported partially by IBM).

An investigation has been started to find improved methods for finding numerical solutions to parameter optimization problems where derivatives of the performance index are not readily computable. Preliminary work indicates that methods more efficient than those presented by Powell can be found.

References

1. M. J. D. Powell, "An Efficient Method for Finding the Minimum of a Function of Several Variables Without Calculating Derivatives," The Computer Journal Vol. 7 (1964), pp. 155-162.
5. Filtering for Multi-Stage Systems with Correlated Measurement Noise. L. Henrikson, Contract Nonr-1866(16). (Supported partially by Dynamics Research Corp.)

Greatly improved methods have been found for designing filters for the title problem. They remove ill-conditioning problems

associated with the augmented-state approach of Kalman, and result in filters of lower dimension, making them more practical. One of the principal applications is to filter data from inertial measuring units.

References

1. R. Kalman, "New Methods in Wiener Filtering Theory," Proc. First Symp. on Engineering Applications of Random Functions Theory and Probability, J. Wiley and Sons, J. L. Bogdanoff, and T. Kozin, Editors, pp. 270-388, 1963.
6. Perturbation Feedback Control with Variable Final Time. J. Speyer and A. E. Bryson, Jr., Contract Nonr-1866(16). (Partially supported by the Raytheon Co. and NASA-ERC).

A very promising improvement in methods for the title problem has been found which may extend the utility of perturbation feedback control to much larger disturbances when the final time is not specified. An intensive program is underway to check out this improvement on realistically complicated problems.

7. Numerical Solution of Optimal Programming Problems with Inequality Constraints. J. Speyer and A. E. Bryson, Jr., Contract Nonr-1866(16). (Partially supported by the Raytheon Co. and NASA-ERC).

An "arc separation" method has been found that promises to greatly simplify the numerical solution of the title problem. Application to a maximum range problem for re-entry gliders is now being made.

References

1. J. P. M. Schalkwijk and T. Kailath, "A Coding Scheme for Additive Noise Channels with Feedback - Part I: No Bandwidth Constraint, " IEEE Trans. on Information Theory, Vol. IT-12, pp. 172-189, April 1966.
2. J. P. M. Schalkwijk, "A coding Scheme for Additive Noise Channels with Feedback - Part II: Band-Limited Signals, " IEEE Trans on Information Theory, Vol. IT-12, pp. 179-189, April 1966.
3. J. K. Omura, Electronics Research Lab. Report, Stanford University, 1966.

8. Optimum Communication with a Feedback Channel . R. B. Campbell and A. E. Bryson, Jr., Contract Nonr-1866(16). (Partially supported by the Raytheon Co.).

The optimum one-dimensional feedback communication system for the additive noise channel has been obtained by use of multi-stage optimization. Optimum, here, is defined to mean that linear system producing a minimum variance unbiased estimate of the transmitted message, subject to an average power, but no peak power, constraint.

Noiseless feedback of the state of the receiver is assumed, and the transmitter uses this information to determine what to transmit. The forward channel is modeled by a discrete-time equivalent of the additive white noise, continuous-time channel.

The problem is couched in state variable notation, and this notation makes it clear that the problem is essentially one in time-discrete optimal control. To be found is the (time-varying) feedback gain in the equivalent control problem; the situation is complicated, however, by the fact that the noise in the equivalent problem depends upon the feedback gain.

The deterministic communication scheme presented here attains capacity in the band-limited and in the infinite bandwidth cases; in fact, the error probability for the additive Gaussian noise case is asymptotically proportional to $\exp - \{ [3/2] \exp 2 (C-R)T \}$ where C is the channel capacity, R is the transmission rate, and T is the coding delay. The double exponential indicates the fundamental capability of the system.

III. C. Information and Control Processes

1. Control Theory and Mathematical Economics. Y. C. Ho, Contract Nonr-1866(16).

Work has resumed on this subject. The problem of optimal investment where investment decisions can only be controlled indirectly through interest rate is investigated. This problem has given rise to phenomena of interest in singular optimal control theory. Effort is being continued.

2. Stochastic Games. Y. C. Ho and R. Behn, Contract Nonr-1866(16).

No significant activity took place on this topic during the past six months. Research on stochastic games is continuing.

References

Colin C. Blaydon, "Approximation of Distribution and Density Functions,"
Proc. IEEE February 1967.

3. Pattern Classification Algorithms. Y. C. Ho and C. C. Blaydon,
Contract Nonr-1866(16).

The work on recursive classification algorithms has been essentially completed. They can be viewed as descent algorithms in noisy environments. Application of these algorithms to control, estimation, and classification problems has been made. A detailed report is forthcoming.

4. Stochastic Descent Algorithms. P. M. Newbold, Contract Nonr-1866(16).

In the same way that optimal control problems interpreted as constrained optimization problems may be solved iteratively by the steepest descent method provided certain conditions are met, the stochastic approximation method can provide the iterative solution to the corresponding stochastic problem subject to some conditions. The similarity between the deterministic and stochastic optimization problems has been explored by Kushner¹.

For the discrete-time linear quadratic problem it has been ascertained that the convergence conditions are identical in form to those given by Gladyshev². In the general case it has merely been shown that the expected costs at each iteration form a non-increasing sequence; thus the cost is a supermartingale and convergence w. p. 1. is implied.

Computer results on a rocket trajectory problem have shown the general validity of the method. It is hoped that extensions to other iterative methods of solution may prove feasible.

- [1] H. J. Kushner, "On Stochastic Extremum Problems," Part I, Journal of Mathematical Analysis and Applications Vol. 10, No. 2, April 1965.
- [2] E. G. Gladyshev, "On Stochastic Approximation," Theory of Probability and Its Applications, Vol. X, No. 2, 1965.

III. D. Topics in Automatic Control

1. Frequency Domain Stability Criteria in Nonlinear Automatic Control. C. P. Neuman, Contract Nonr-1866(16) .

An experimental and theoretical study of the frequency domain stability criteria in nonlinear automatic control has been completed. A technical report describing this work is in preparation.

IV. COMMUNICATIONS AND NETWORKS

Personnel

Asst. Prof. D. W. Tufts	Mr. H. Gish
Dr. A. A. Pandiscio	Mr. J. Proakis
Dr. P. J. Trafton	

IV. A. Electronics and Communications

1. Digital/Analog Multiplier. A. A. Pandiscio, Contract Nonr-1866(16).

An important element of hybrid systems is the digital/analog multiplier. As the name implies such a device forms the four quadrant product of an analog signal and a signed digital number. As such it is an integral part of hybrid computation systems and can also be very useful in information display systems driven by a digital computer. This project has just started and will be directed towards the development of such a multiplier. Present indications are that the input current switching section can best be realized using integrated circuits while the current summing amplifier can be best realized using a discrete component operational amplifier.

2. High Speed Photoformer. A. A. Pandiscio, Contract Nonr-1866(16).

A redesign has been undertaken of the photoformer previously developed for use in experimental studies in communication theory. This redesign has as its objective higher speed capabilities than presently exist and self-contained operation not requiring that a general purpose oscilloscope be tied up.

IV. B. Communications Theory

1. Data Transmission by Pulse Amplitude Modulation Through a Noisy Channel which has been Randomly Selected. D. W. Tufts, Contract Nonr-1866(16) .

A criterion of minimum mean square error is used to specify the optimum, linear, time-invariant equalizer for a system in which data is transmitted by pulse amplitude modulation through a randomly selected channel. The results are applied to the case in which the received pulse is a synchronous demodulation of a passband pulse, with the phase of the demodulating oscillator a random variable and with a random time delay.

The results can be applied to automatic equalization, because it is desirable to reduce the sensitivity of the equalizer's performance to residual errors.

2. Modulo-m Linear Sequential Circuits, Partial Response Signaling Formats, and Signal Flow Graphs . D. W. Tufts, Contract Nonr-1866(16) .

We develop signal flow graph techniques for the analysis of modulo-m linear sequential circuits and of mixed systems containing both modulo-m adders and adders of real numbers. One of our main results is the proof that the rational generating function of a modulo-m linear sequential circuit can be obtained from the generating function of a topologically similar sampled data system (with integer branch gains) by simply replacing each integer valued coefficient in the latter generating function by its (mod m) equivalent.

Besides the direct application to the analysis of modulo- m sequential circuits and sequence generators we also investigate mixed systems which arise in the partial response signaling format for digital data transmission.

3. Adaptive Receivers for Digital Signaling Over Random or Unknown Channels . J. Proakis and D. Tufts, Contract Nonr-1866(16) .

The problem in digital communications treated herein can be stated as follows: There is a transmitter which chooses and sends in any signaling interval of duration T seconds, one of M possible signaling waveforms over a channel which is modeled as a linear filter with an impulse response that is unknown or randomly time-varying. The transmitted waveform is also corrupted by additive Gaussian noise. At the receiving point a receiver is to be designed which processes the received signal and decides with minimum probability of error which one of the M waveforms was transmitted in every signaling interval.

Treated in detail is a channel which imparts an unknown or slowly and randomly time-varying gain and phase shift to the signal which is transmitted in each signaling interval. For this channel the receiver which minimizes the probability of error is specified and its performance is evaluated for binary orthogonal signaling and M -phase signaling. Basically, the receiver uses an estimate of the channel gain and phase shift in detecting the waveform received in each signaling interval. Furthermore, it is shown that in addition to detecting the transmitted information sequence, this receiver continuously estimates the channel gain and phase shift either from the information-bearing waveform or by means of a pilot waveform which is transmitted for the purpose of

measuring the channel.

The performance of the receiver is evaluated in terms of its probability of error for binary orthogonal signaling and M-phase signaling in the general context of Lth order diversity reception. For M-phase signaling we give the probabilities of a binary digit error and the probability of a character error.

For more general linear channel models of unknown time-invariant channels or randomly time-varying channels, intersymbol interference tends to limit the performance of the receiver and, hence, this interference becomes an important consideration in the design of the receiver. Here we specify a receiver which minimizes the effects of intersymbol interference either in conjunction or independently of the additive Gaussian noise. In this case the receiver takes the form of an equalizer which is synthesized on the basis of estimates of the channel response to a sounding signal waveform. The performance of the receiver is difficult to evaluate for general channel models. However, by means of Monte Carlo simulations performed on a digital computer we obtain the probability of a binary digit error for the equalizer in the case of a multipath, randomly time-varying channel with an impulse response which is modeled as consisting of three statistically independent paths.

4. Quantization and Reconstruction of Random Sequences . H. Gish,
Contract Nonr-1866(16) .

Delta modulation and Differential PCM systems are information transmission systems which employ quantizers with some sort of feedback around the quantizer. The systems being investigated are those in which linear quantizer feedback and linear receivers are

employed. Recent research has shown that digital communication systems which employ a quantizer with feedback can provide superior performance than PCM, which is a widely used digital system which employs a zero memory quantizer.

5. Band-Pass Limiter Envelope Distributions and Statistics .

D. W. Tufts and P. Trafton, Contract Nonr-1866(16) .

We develop some methods for approximate prediction of statistics and probability distributions associated with the output $z(t)$ of the detection system sketched in Fig. 1 . In developing our approximations we utilize basic results of Rice [1] and Rayleigh [2] as refined and extended by Davenport and Root [3] and by Doyle and Reed [4]. In overcoming experimental difficulties we have been encouraged by the successful efforts of Bogotch and Cook [5], who have obtained interesting results for a similar problem.

The system of Fig. 1 is intended to decide whether or not the signal $s(t)$ is present at its input by comparing the value of its output with a threshold value at a given instant of time. If the threshold value is exceeded, then the signal is said to be present. Throughout our discussion the input noise is Gaussian and white over the band of the input filter and the interference and signal are sinusoids of different frequencies. The signal frequency is centered in the pass band of the postfilter, as shown in Fig. 2 . For much of our work, a sinusoid at the interference frequency is assumed to be strongly attenuated by the postfilter.

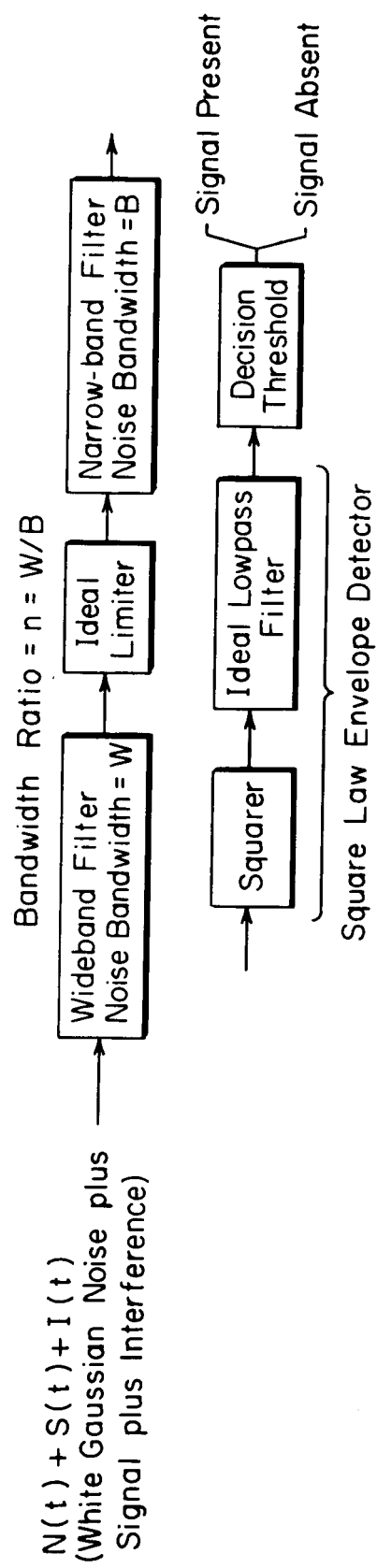


FIG. 1 BLOCK DIAGRAM OF A BANDPASS LIMITER PLUS SQUARE LAW DETECTOR.

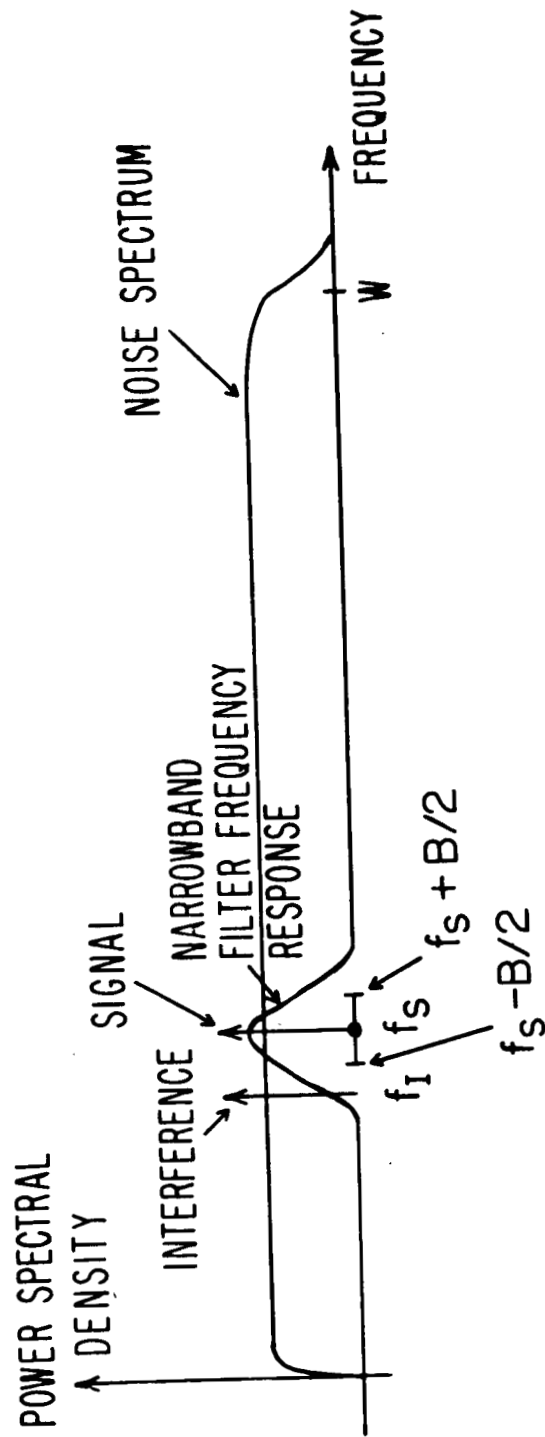


FIG. 2 POWER SPECTRA OF SIGNAL, INTERFERENCE, AND NOISE AT THE LIMITER INPUT.

From the detection standpoint, the presence of the limiter in the system of Fig. 1 has two advantages. First, for any fixed ratios of signal, interference, and noise powers, the probability distribution of the output random variable $z(t)$ is independent of input level changes (i.e., scaling factors introduced before the limiter). And second, the limiter transforms its input into a normalized form which is convenient for further processing.

References

- [1] Rice, S. O., "Mathematical Analysis of Random Noise," Bell Systems Tech. Journal, 24, January 1945, pp. 115-162.
- [2] Lord Rayleigh, "On the Problem of Random Vibrations, and of Random Flights in One, Two, or Three Dimensions," Phil. Mag. 37, 1919, pp. 321-347. (This paper is reprinted in "Scientific Papers of Lord Rayleigh.")
- [3] Davenport, W. B., and Root, W. L., "Random Signals and Noise," McGraw-Hill Book Company Inc., 1958, pp. 158-167 and 250-260.
- [4] Doyle, W. and Reed, I. S., "Approximate Band-Pass Limiter Envelope Distributions," IEEE Transactions on Information Theory, Vol. IT-10, July 1964, pp. 180-185.
- [5] Bogotch, S. E. and Cook, C. E., "The Effect of Limiting on the Detectability of Partially Time-Coincident Pulse Compression Signals," IEEE Transactions on Military Electronics, Vol. MIL-9, January 1965, pp. 17-24.

N67-32220

V. MICROWAVE APPLICATION OF FERROMAGNETIC AND
FERROELECTRIC MATERIALS

Personnel

Prof. R. V. Jones	Mr. F. A. Milton
Mr. R. Bartkowski	Mr. J. P. Sage
Mr. P. Berger	Mr. R. Tancrell
Mr. B. Levine	

1. NMR Measurements in Magnetic Garnets . R. Tancrell, Contracts Nonr-1866(16) and AF 19(628)-3874.

The NMR of gallium impurities in YIG represents an interesting example of a local mode resonance. The nonmagnetic gallium impurity replaces a ferric ion on a crystallographic d site creating a defect in the ordered magnetic structure. However, the nuclear moments of the abundant Ga and Ga nuclei are coupled to the surrounding magnetic ions by a transferred hyperfine interaction which is presumably of the same form, i. e., $A \cdot S$ -as the Heisenberg interaction between the ions of the host lattice. Thus, from a formal point of view, the gallium nuclei represent spin impurities coupled to the surrounding spins by a greatly reduced effective exchange interaction. The theoretical description of this situation is quite distinct from that of more common cases of NMR in ordered magnetic systems since here the nucleus forms an intrinsic part of the local defect.

The impurity states of a magnetic impurity in a Heisenberg ferromagnet have been treated theoretically in considerable detail and

these considerations may be applied, with due circumspection, to this problem. In particular, Izyumov and Medvedev* have derived expressions for the excitation spectrum for a magnetic impurity with a g factor different from that of the host ion. Their results are valid even though the g value and effective exchange constant are significantly different from those of the host, provided only that the range of the perturbation is short. In the limit of very small impurity concentrations and for a finite applied magnetic field, each gallium ion contributes one mode to a localized impurity level or state lying well below the spin wave band of the undisturbed crystal. From a more conventional view point, this local mode represents, of course, the NMR mode at a frequency $A(S)/h$. It is our view that recasting this NMR problem in the language of impurity states is not merely a semantic exercise, but rather the means of systematizing a difficult problem. For example, Izyumov and Medvedev have given susceptibility expressions for the response of the local mode to an oscillatory magnetic field. The character of this response is sensitive to the disposition of the local mode with respect to the spin wave band and gives a direct picture of the enhancement effects of NMR in this material.

The local mode picture is perhaps most useful in the discussion of the line breadth and relaxation characteristics of the NMR. Spin echo experiments have been performed in the helium temperature range on both mono- and polycrystalline samples of $Y_3Fe_{5-x}Ga_xO_{12}$. Perhaps

* Y. A. Izyumov and M. V. Medvedev, Soviet Physics, JETP 21, 1155 (1965).

the most remarkable characteristic of these observations is the extreme breadth of the resonance lines in single crystals and for small Ga concentrations. For example, a sample with $x = 0.25$ gives for the magnetization in the $[111]$ direction a width of 5 MHz for the Ga line at 26 MHz. The quadrupole interaction can, at most, account for 1.5 MHz of this width. In contrast the homogeneous width, as measured by the coherence time T_2 , is only about 12 Hz. The peculiar shape of the observed echo is a further indication that the line is inhomogeneously broadened. We attribute the extreme breadth of the line to the strong interaction between localized impurity states. Lifshitz has discussed the broadening of the impurity mode as the interaction between impurities is taken into account. This broadening is not associated with the reduction in the lifetime of the state but rather a broadening of the spectral distribution of the impurity states. The observed lineshape for single domain samples is similar to that in powders. In comparison, the Fe line of the host ion is found to be much narrower in width for a given Ga concentration and to increase steadily as the Ga concentration increases. It seems that the localized states are more sensitive to the range of the interaction between impurities and may show significant narrowing at concentrations smaller than $x = 0.25$. Fortunately, because of signal to noise problems we have not been able to measure the linewidth at smaller concentrations.

The longitudinal relaxation T_1 has been measured by observing the stimulated echo following a three-pulse sequence. The echo decay was not exponential, although the initial fall to half the initial height exhibited a constant rate of decrease. When the external field is zero or very small, the impurity modes are degenerate with the spin waves and domain wall excitations. In this situation the local mode becomes a virtual state

of finite lifetime. Thus, as might be expected the observed T_1 is quite short when domain walls are present and increases rapidly as the sample is magnetized. T_1 is extremely concentration dependent. In addition T_1 is found to vary as T^{-1} between 1.9° and 4.2° K and to increase by a factor of two as the drive frequency is changed from 27 to 19 MHz. These results suggest that the dominant process is a direct one in which the nuclei relax through a phonon modulation of the interaction between impurities. For the direct process T_1^{-1} varies directly as the temperature and the square of the energy separation as observed.

2. Ferromagnetic Relaxation in Metals . A. F. Milton, Contracts Nonr-1866(16) and AF19(628)-3874 .

A systematic understanding of the relaxation mechanisms in a ferromagnetic metal is not conveniently available. Both the theoretical and experimental pictures are cloudy. The basic theoretical problem is to determine how the collective magnetic excitations -spin waves- lose their energy to the single particle excitations of the electronic system. In the iron group metals, this process undoubtedly involves an interaction between the more itinerant s-electrons and the more localized d-electrons. However, the details or magnitude of this interaction are far from clear.

On the experimental side, it has been very difficult to get any notion of the size of intrinsic relaxation times. As yet, inelastic neutron scattering experiments do not have sufficient precision to give meaningful measurements. The interpretation of ordinary ferromagnetic resonance (FMR) in metals is greatly hampered by "skin depth" effects. In particular, surface imperfections cause gross inhomogeneities in resonance line shapes and "exchange-conductivity" effects lead to a dispersive line broadening. We have been attempting to circumvent these difficulties by

using the parallel pump instability (PPI) technique. This technique and our early work in metals have been described in Technical Report No. 485 .

Our early work concentrated on metallic thin films. The more recent work has been on bulk single crystals. A fairly consistent picture of long wavelength spin wave relaxation now begins to emerge, at least for nickel-iron alloys. The relaxation rates in these alloys are at least an order of magnitude greater than the rates in ferromagnetic insulators. The following table summarizes some of our preliminary data on permalloy for spin waves at a frequency of 8Gc:

<u>Alloy(Ni i Fe)</u>	<u>Temperature</u>	<u>Equivalent Width</u>	<u>Method</u>
81:19 (Bulk)	77°K	85 oe	PPI
81:19 (Bulk)	200°K	78	PPI
81:19 (Bulk)	300°K	60	PPI
83:17 (Film)	300°K	85	PPI
83:17 (Film)	300°K	~ 50	FMR
70:30 (Film)	300°K	90	PPI
75:68 (Film)	300°K	68	PPI
80:20 (Film)	300°K	61	PPI
87:13 (Film)	300°K	95	PPI

3. Ferromagnetic Relaxation in Magnetic Oxides . R. Bartkowski and J. P. Sage, Contract Nonr-1866(16).

The spinel, insulator $\text{Cd Cr}_2 \text{Se}_4$ has been recently found to be ferromagnetic. The rather simple magnetic structure of this substance makes it an attractive substance for the study of magnetic relaxation mechanisms in general, but in particular those mechanisms of

importance in the vicinity of the ordering temperature. We have studied the ferromagnetic relaxation of CdCr_2Se_4 by means of both FMR and PPI. Using the latter technique, the intrinsic spin-wave linewidth has been measured up to temperatures within a few degrees of the Curie temperature. At low temperature (below approximately 100°K) the PPI derived linewidth is essentially insensitive to temperature. This is a surprising result, since one would expect all of the intrinsic relaxation mechanisms to be strongly temperature dependent. However, near the Curie temperature the linewidth seems to diverge as $(T_c - T)^{-1}$ with $T_c = 137^\circ\text{K}$. This value of T_c is in close agreement with our magnetic data. We do not, as yet, understand this simple behavior. The behavior of the linewidths derived from the two techniques differ in several significant ways. For example, the FMR width has some appreciable crystalline anisotropy while the PPI is isotropic.

4. Ultrasonic Attenuation in Magnetic Oxides . B. Levine, Contract Nonr-1866(16).

The orthorhombic phase of the compound GaFeO_3 (abbreviated here as GIO) is unique in that it is the only currently known piezoelectric material which has an appreciable spontaneous magnetic moment. The coexistence of these two attributes necessarily implies the presence of several interesting forms of "magnetoelectric" interactions.

In order to understand better any possible applications, a systematic study of the ultrasonic propagation in GIO has been undertaken. In particular, we are interested in studying any magnetic field induced

dispersions and attenuations. The work over the last few months has been largely devoted to developing techniques for exciting and detecting ultrasonic waves. However, considerable effort has also been directed towards the development of a theoretical model of the magnetic properties of GIO .

VI. ELECTROMAGNETIC PHENOMENA

Personnel

Prof. R. W. P. King	Mr. B. M. Duff
Prof. T. T. Wu (on leave)	Mr. S. Holly
Asst. Prof. B. Rama Rao	Mr. C. C. Kao
Dr. C. L. Chen	Mr. D. Lamensdorf
Dr. K. Iizuka	Mr. T. Padhi
Dr. R. B. Mack	Mr. J. C. Robertson
Dr. S. S. Sandler	Mr. A. E. Sanderson
Dr. W. A. Saxton	Mr. L. D. Scott
Dr. S. R. Seshadri	Mr. L. C. Shen
Dr. H. S. Tuan	Mr. R. Shore
Dr. Y. S. Yeh	Mr. T. L. Simpson
Mrs. D. G. Tingley	Mr. T. Sugimoto
Mr. D. C. Chang	Mr. C. Y. Ting
Mr. V. W. Chang	Mr. A. D. Wunsch
Mr. W. M. Cheong	

A. ANTENNAS

Researchers in progress in the field of single antennas are concerned with several problems that are both of practical importance and scientific interest. The first of these is the cylindrical antenna with cross-sectional dimensions that are comparable with the wavelength. Such antennas are useful as structurally rigid microwave elements, as broad-band antennas, in problems in which bombs and missiles are treated as radiating elements, and in applications in which tubular structures are used as antennas. The conventional theory of the antennas is not applicable and new theoretical and experimental techniques are required. A second group of studies is concerned with directional broad-band antennas that can be deployed from a satellite. For this purpose traveling-wave resistive and resistively-loaded elements are being investigated. Other researchers deal with thin-wire antennas of the top-loaded

type - inverted L, T and umbrella structures - which are useful in low-frequency transmission, with electrically long dipoles and loaded loops.

1. Theory of the Electrically Thick Antenna . R. W. P. King and T. T. Wu, Contract Nonr-1866(32).

A paper entitled, "The Thick Tubular Transmitting Antenna, " by R. W. P. King and T. T. Wu has been submitted to Radio Science for publication.

2. The Thick Cylindrical Antenna. D. C. Chang, Contracts Nonr-1866(32) and AF19(628)-2406.

The experimental investigation of an electrically thick cylindrical antenna is in progress. The purpose of this investigation is to verify the theoretical analysis of the same problem, solved numerically by an "approximate product-integration" technique. (Chang, Cruft Technical Report No. 509). The main features of the experimental set-up are: (1) the antenna is driven by a coaxial-line which, in turn, is excited by two current loops located near the other end of the coaxial-line. The two loops are located diametrically opposite, and are tuned to have currents in the same amplitude and the same phase. At the end of the coaxial-line, a movable short-circuit is inserted, so that the undesired, yet propagating TE_{11} mode may be tuned out. (2) The current distribution and input admittance of the antenna can both be measured from a small loop which slides up and down in a tube $1/8$ " in diameter. This small tube is attached to the surface of the inner conductor which is 6" in diameter. Unlike the conventional set-up, the whole system will have no undesired radiation from slots.

3. Experimental Study of Electrically Thick Antennas. S. Holly,
Contract AF19(628)-2406.

A final report on this project is in preparation.

4. Infinitely long Electrically Thick Antennas. R. Shore, Nonr-1866(32).

The study of the thick dipole antenna is important for broad-banding applications. As an approach towards an understanding of the thick dipole, an analysis has been performed of an infinite dipole antenna of arbitrary radius driven from a coaxial line. Both the rotationally symmetric TEM mode and the TE_{11} mode of excitation have been considered. The solutions are exact and are obtained by the use of the Wiener-Hopf technique. Expressions have been obtained for the reflection coefficients, gain, and asymptotic current distributions. Numerical results are in the process of being obtained.

5. Theoretical Study of the Resistive Antenna. R. W. P. King and
T. T. Wu. Contract Nonr-1866(32) and NASA Grant NsG-579.

Papers entitled, "The Imperfectly Conducting Cylindrical Transmitting Antenna," by R. W. P. King and T. T. Wu and "The Imperfectly Conducting Antenna: Numerical Results," by R. W. P. King, C. W. Harrison Jr., and E. A. Aronson have been published in Transactions of IEEE, Vol. AP-14, pp. 524-543, September 1966.

6. Theoretical and Experimental Studies of Resistive Antennas .
L. C. Shen, NASA Grant NsG-579.

The dipole antenna with nonreflecting resistive loading has been studied experimentally. The amplitude of the current, the input admittance, and the radiation field pattern were measured in the frequency range 450 to 900 MHz. Four different designs of the resistive loading

were tried in order to simulate the antenna with the smoothly distributed loading which has been studied theoretically in the previous NASA reports. Six antennas were constructed and the experimental data were obtained to be compared with the theory. It has been found that the zero-order theory gives accurate descriptions of the relative current distribution, the field pattern, the property of a very broad frequency band of the antenna, and the existence of the traveling wave on the antenna. It was noted that such comparison was meaningful in the light of the analysis carried out in the previous report (Scientific Report No. 6). The quantitative agreement between the zero-order input admittance and the measured value is not good, but with the help of the variational principle it was found that a simple trial function for the current distribution could be used to improve the theory. A Scientific Report on this investigation has been issued: Scientific Report No. 7 NsG-579, "An Experimental Study of the Dipole Antenna with Non-reflecting Resistive Loading," by Liang-Chi Shen, September 1966. A supplement to Scientific Report No. 7 titled, "A Comparison between the Variational Solution and the Experimental Data," by Liang-Chi Shen was issued later.

Scientific Report No. 5 under Grant NsG-579, "The Cylindrical Antenna with Tapered Resistive Loading," by Liang-Chi Shen and Tsi Tsun Wu, has been accepted by Radio Science for publication; it is scheduled to appear in the February issue of 1967.

7. Traveling Wave V-Antenna. K. Iizuka, NASA Grant NsG-579.

A report describing a study of the current distribution on a sphere on which a traveling wave V-antenna is mounted is in preparation.

8. Thin-Wire and Electrically Short Antennas. T. Simpson,
Contract Nonr-1866(32). (Partially supported by DECO Communications).

The analysis of a representative set of top-loaded antennas was undertaken to supplement the extensive experimental work in this area. The first problem studied was the "Inverted L" Antenna i.e., an antenna with one top-loading member at right angles to the vertical member. The analysis was carried out using numerical techniques to solve the simultaneous integral equations for the currents. After verification was obtained from experimental models, the method was successfully extended to the "T" and "Turnstile Top-Load" Antennas. Measured admittances of these antennas were obtained over a wide range of electrical heights from well-below first resonance to above anti-resonance. These results, together with measured current and charge distributions tend to confirm the adequacy of the theoretical approach. At the outset of this work it was expected that the inadequacy of the one-dimensional theory in the junction region (e.g., the corner of the inverted L) would lead to some discrepancy in the results. Comparison between theory and available and experimental results in the critical region about resonance exhibit some evidence of this difficulty. However, the discrepancy may be interpreted as a difference between the physical center-line length of the bent conductor and an apparent electrical length of the actual model. From this viewpoint the discrepancy is observed to be of the order of the conductor radius. Since the one dimensional theory is only valid for antennas of small radii, the error involved is also quite small.

Presently, the theory is being extended to handle a more general structure involving top-loading members which make an arbitrary angle, θ , with the vertical member. Models have been constructed with

$\theta = 45^\circ$, the so-called "Umbrella Antenna," and will be tested and compared with the theoretical results when these become available.

9. Long Dipole Antennas. C. L. Chen, R. W. P. King and S. S. Sandler, Contract Nonr 1866(32) and NSF Grant GK-273.

A report on an experimental and theoretical study of the long receiving dipole is in preparation. A study is being made of the possibility of representing the distribution of current in long transmitting dipoles by simple trigonometric functions both when the antennas are isolated and coupled. If successful it should permit an approach to the approximate treatment of transients in coupled antennas

10. Studies on Loaded Loop Antennas. K. Iizuka, Contract Nonr-1866(32).

A technical report on studies made on an array of circular loops loaded with tunnel diodes is in preparation.

The construction of a wireless probe to measure the current on a parasitic antenna has been started.

B. COUPLED ANTENNAS AND ARRAYS

As a first step in the theory of the response of coupled antennas and arrays to pulses and transients the steady-state properties of two parallel electrically very long dipoles are being studied. A beginning is also being made on understanding the properties of coupled electrically thick antennas which are involved in structurally rigid directional couplets and arrays. The general study of arrays (which takes

complete account of the mutual interaction of the currents in different elements both in affecting the distributions of current and the admittances) has been extended to include arrays of unequal and unequally-spaced elements, notably a complete analysis of the important frequency independent log-periodic dipole array has been achieved. An accurate treatment of collinear and staggered elements and of planar and three-dimensional arrays is in progress.

11. The Theory of Long Coupled Antennas. T. Padhi,
Contracts Nonr 1866(26) and Nonr 1866(32).

The self and mutual admittances of an array of two long, thin, identical dipole antennas have been determined by an approximate method based on the Wiener Hopf technique when the antennas are center driven, parallel and non-staggered. Part of the analysis has appeared in Cruft Lab Tech Report No. 511. It has been found possible to obtain results for the steady state without recourse to numerical integration. The use of these results for the calculation of the transient response is at present under study.

12. Experimental Study of Two Parallel Circular Arrays and of Two Parallel Electrically Thick Antennas. B. M. Duff,
NSF Grant GK-273.

Reports are in preparation to describe extensive experimental and theoretical studies on the properties of two coupled circular arrays and two coupled cylinders with cross sections comparable with the wavelength.

13. Theoretical and Experimental Studies on Unequal and Unequally-Spaced Dipoles. W. M. Cheong, NSF Grant GK-273

Research on this subject has been now completed. The measurements made on a five-element log-periodic dipole antenna using a coaxial-line feeding arrangement have yielded results that agree excellently with theory.

Also, a twelve-element log-periodic dipole antenna has been investigated thoroughly using the modified three-term current theory. Some very important properties of the antenna have been brought to light. It is now possible to explain the operation of the log-periodic dipole antenna on a firm theoretical basis. Two papers entitled "Arrays of Unequal and Unequally-Spaced Dipoles" and "The Log-Periodic Dipole Antenna" by W. M. Cheong have been submitted for publication.

14. Planar and Three-Dimensional Arrays. V. W. Chang, NSF Grant GK-273.

A dipole array of N-parallel arbitrarily located antennas is being investigated. In the case of two parallel antennas in echelon or in a collinear arrangement, the integral equations are first decoupled into two independent integral equations, and then solved in two different methods, a numerical method and an approximate method in terms of a five-term representation of the current. The five-term approximate theory is an extension of King's three-term theory. The five-term approximate theory is further extended to apply to arrays of N-parallel dipole antennas. Both the current distribution and the far field pattern are being investigated as is the effect of

beam steering. Computed results of a typical two-dimensional 3×3 planar array and a typical three-dimensional $3 \times 3 \times 3$ array have been obtained. Two technical reports, one dealing with the two-element array and the other with the N-element array, are being prepared.

15. A Study of Arrays of Dipole Antennas. R. W. P. King, R. B. Mack and S. S. Sandler, Contracts Nonr-1866(32) and AF 19(628) -2406 and NSF Grant GK-273.

The manuscript for a book entitled "Arrays of Cylindrical Dipoles" by R. W. P. King, R. B. Mack and S. S. Sandler is in press with the Cambridge University Press. In its final form the manuscript consists of eight chapters entitled 1. Conventional Theory of Antennas and Arrays 2. An Approximate Analysis of the Cylindrical Antenna 3. The Two-Element Array 4. The Circular Array 5. The Circuit and Radiating Properties of Curtain Arrays 6. Arrays with Unequal Elements: Parasitic and Log-Periodic Antennas. 7. Planar and Three-Dimensional Arrays 8. Techniques and Theory of Measurement.

16. Theoretical Investigation of the Current Distribution In An Array of Circular Loops. C. C. Kao, Contract AF19(628)-2406.

The problem of an array of N coupled circular loop antennas was investigated. The loops are arranged to be parallel and concentric. The loop radii kb_i , the wire radii ka_i and the spacings kb_{ij} are arbitrary. The distribution of current in each element can be represented in either of the following two forms:

(i) if $kb_i \neq \text{integer}$

$$I_i(\theta) = \frac{-j2\pi}{\xi} \left[I_{vi}(\sin kb_i |\theta| + \cot kb_i \pi \cos kb_i \theta) + \frac{I_{i,0}}{2} + \sum_{m=1}^{\infty} I_{i,m} \cos m\theta \right]$$

(ii) if $kb_i = \text{integer}$

$$I_i(\theta) = \frac{-j2\pi}{\xi} \left[I_{vi}(\sin kb_i |\theta| - \frac{\theta}{\pi} \sin kb_i \theta) + \frac{I_{i,0}}{2} + \sum_{m=1}^{\infty} I_{i,m} \cos m\theta \right]$$

where I_{vi} depends on the driving voltage V_i . For each m , the current coefficients $I_{i,m}$ ($i = 1, 2, \dots, N$) in the series are found coupled together and can be easily obtained by solving a set of N linear simultaneous equations.

The special cases of the isolated loop ($N = 1$) and of two coupled identical loops ($N = 2$; $kb_1 = kb_2$, $ka_1 = ka_2$) were calculated. In these, the series were truncated at $m = 3$ or 4 . The comparison with other solutions in the literatures was very satisfactory.

C. ANTENNAS AND WAVES IN DIELECTRIC CONDUCTING AND PLASMA MEDIA

Investigations in progress on the properties of antennas when immersed in homogeneous, stratified, or anisotropic media can throw light on problems in communication from or into the earth or the sea, geophysical exploration including the detection of ore deposits and buried conductors, and in laying a foundation for the study of antennas in plasmas. At the recent international assembly

of URSI it was emphasized that past research in the field of plasmas has been largely in the solution of theoretical problems of an academic type not susceptible to experimental verification. It was recommended that a major effort be made to coordinate theoretical and experimental research. The studies listed below are designed for this purpose. Experimental and theoretical studies are being coordinated in an effort to determine the properties of the plasmas and of the antennas and other probes used to interact with them.

17. Antenna in Conducting Half Space. H. S. Tuan and R. W. P. King
Contract Nonr 1866(32).

A paper entitled "Current in a Scattering Antenna Embedded in a Dissipative Half Space" by H. S. Tuan and R. W. P. King has been published in Radio Science, Vol. 1. P. 1306, Nov. 1966.

18. An Experimental Study of the Properties of Antennas When Immersed in Conducting Dielectrics. K. Iizuka and
T. Sugimoto, Contract Nonr-1866(32).

A paper entitled, "A Technique of Fabricating an Inhomogeneous Medium and the Behavior of a Dipole in Such a Medium," was accepted for publication in the Proceedings of IEE (London). This paper discusses an antenna immersed in a medium with the gradient of the loss tangent perpendicular to the axis of the dipole. Measurements have been made of the properties of a dipole antenna in a medium in which the gradient of the loss tangent is parallel to the axis of the dipole.

The papers, "Measurement of Dielectric Properties of

High-Loss, High-Dielectric Soft Materials by Parallel-Plate-Region Method," and "Agar-Agar Chamber for the Study of Electromagnetic Waves in an Inhomogeneous Medium," are in preparation.

The study of a new method of measuring the conductivity of agar agar at lower frequencies by electromagnetic induction has been started.

19. Antennas in a Cylinder of Anisotropic Material . D. Lamensdorf, Contract AF19(628)-2406.

Numerical methods are being used to calculate the effect of the transition region between a coaxial line and a monopole antenna driven over a ground plane. An electrostatic approximation is used with the potential fields of the transition region compared with those of an infinite electrostatic dipole with a voltage delta function at the origin. The difference between the two cases will be represented by a lumped capacitance.

Measurements are being made on a monopole antenna surrounded by cylinders of an artificial anisotropic dielectric. The anisotropy is produced by thin layers of polyfoam and Stipast HiK, a low-loss, high-dielectric constant material. Admittances and currents have been measured for symmetric layers in the axial direction. Similar measurements are planned for planar (i.e., asymmetric layers in the radial direction).

20. Theoretical and Experimental Study of Finite Cylindrical Antennas in a Plasma Column. C. Y. Ting and B. Rama Rao, NASA Grant NGR-22-007-056 and Contract Nonr 1866(32).

By means of the Wiener-Hopf technique, the reflection coefficient of the transmission current at the end of a dielectric-coated antenna can be expressed in a single integral form. This result, when used with the solution of an infinite dielectric-coated cylindrical antenna, yields the input admittance and the current distribution of a long dielectric-coated antenna. It is found that unlike the locus of the admittance of a bare cylindrical antenna, which converges to a point as the antenna gets longer and longer, the locus of the input admittance of the dielectric-coated antenna becomes a circle. Also, due to the reflection of the transmission current back and forth a standing wave with wavelength equal to that of the surface wave is formed along the antenna.

An experimental study of the plasma-coated antenna is in progress. It has been found that the microwave-cavity perturbation method is the best way to measure the electron density and the effective collision frequency. The measurement of current distribution and input admittance is in progress.

21. Experimental and Theoretical Investigations on Plasma Coated Antennas. B. Rama Rao and C. Y. Ting, NASA Grant NGR-22-007-056.

Extensive experimental measurements have been made to determine the current-distribution and impedance characteristics of plasma-coated cylindrical dipole antennas with lengths comparable to the free space wavelength. Measurements were made at frequencies

both above and below the plasma frequency. When the excitation frequency of the antenna is above the plasma frequency, the plasma sheath appears to reduce the electric length of the antenna, causing the current distribution to become more uniform. The experimental results show a "reasonable" degree of qualitative agreement with the theoretical results obtained by C. Y. Ting assuming a lossless cold-plasma model (hydrodynamic approach). In the vicinity and below the plasma frequency, the current distribution of the antenna undergoes a very marked change - the hydrodynamic theory seems unable to account for this phenomenon. An alternative approach using the moment equations of the Boltzmann equation (to take into account the temperature effect) seems necessary.

By far the most interesting effect noticed was the sharp increase in the input resistance of a short dipole antenna as the electron density was increased. The input impedance of a short antenna ($\frac{\lambda_0}{32}$) is mainly capacitive, the measured input resistance being only 4 ohms. As the electron density (discharge current) was steadily increased, the input resistance of the antenna rose sharply to 400Ω and then fell off to 200Ω and remained steady thereafter, with further increase in the plasma frequency. This "resonance" phenomenon occurs below the measured plasma frequency; the resonance line appears quite broad. A suitable theoretical explanation for this resistance peaking is being sought by taking into account the resonances in the warm plasma column. Strong resonances in the plasma column could absorb the electromagnetic energy supplied to the antenna thereby causing its radiation resistance to increase sharply. The resonance line observed was quite broad, indicating

the possibility of some type of collisionless damping. The possibility of using short dipole antennas for studying such plasma column resonances is being investigated.

Measurements are now being made to study the effect of the electron temperature in the antenna characteristics. These investigations will be made in the vicinity of the plasma frequency, where the compressibility effects of the plasma should have their maximum impact.

The experimental results will be reported in a forthcoming technical report.

22. Plasma Sheath Investigations Using a Microwave Cavity Technique.

B. Rama Rao and L. D. Scott, NASA Grant NGR-22-007-056.

The cavity perturbation method has been widely used as a diagnostic technique for studying plasma properties. The accuracy of this method depends largely on the assumed electron-density profile distributions used in the calculations. The profile distribution commonly used in this type of measurement is the Schottky diffusion profile of the type $J_0(2.404 \frac{r}{r_p})$, when r_p is the radius of the plasma column. Many investigators fail to realize that this type of profile is only valid at high pressures when the mean free path of the electrons is small compared to the radius r_p of the plasma column; substantial errors can result if the same profile is used for measurements at low pressures or when the plasma column is very thin.

The purpose of this study is to use a multi-mode cavity method so that the plasma diagnosis can be extended over a much wider range of pressures. Subsequently this method will be employed to make an

experimental study of the plasma sheath problem - where the radial density distribution will be studied for a wide range of pressures and electron temperatures. A similar method has been suggested by Stewart⁴, but it has not been used for the specific purpose of diagnostic and sheath investigations.

To simplify the analysis of the problem, three different pressure regimes will be considered.

(1) High pressure regime, (pressure > 1 mm of Hg; mean free path $\ll r_p$; ambipolar diffusion type regime). The differential equation for the electron density with both ambipolar diffusion and volume recombination taken into account may be written in the form¹

$$\frac{d^2 n}{dx^2} + \frac{1}{x} \frac{dn}{dx} + \frac{\nu r_p^2}{D_a} n - \frac{\beta r_p^2}{D_a} n = 0$$

where r_p = plasma tube radius, D_a = ambipolar diffusion constant, ν = coefficient of ionization, and variable $x = r/r_p$. Solution to this equation is of the form

$$n = n_0 + \sum_{\ell=1}^{\infty} \left[\frac{1}{(2\ell)^2} - \frac{\nu r_p^2}{D_a} a_{2(\ell-1)} - \frac{\nu r_p^2}{D_a} \sum_{k=0}^{\ell-1} a_{2k} a_{2(\ell-k-1)} \right] x^{2\ell}$$

Hence in this regime the electron density profile is of the form

$n = n_0 - n_2 x^2 + n_4 x^4$. Only the first three terms need be considered since the above series is rapidly convergent. The next step in the experimental technique is to measure the shift in the resonant frequency using three different cavity modes with different electric field distributions along the radial directions. For the right circular cylindrical cavity used in the experiments the three modes chosen were the TM_{020} , TM_{120} , and the TM_{210} . These have only E_z components

which vary respectively as $J_0\left(\frac{5.52r}{r_p}\right)$, $J_1\left(\frac{7.016r}{r_p}\right)$, and $J_2\left(\frac{5.136r}{r_p}\right)$. By measuring the frequency shifts of these three

modes, the profile coefficients n_0 , n_2 , and n_4 can be calculated. n_0 is the electron density at the axis of the plasma column. Additional checks were made noting the TM_{012} and the TM_{212} modes. The measurements were made on a hot-cathode discharge tube using helium.

(2) Intermediate pressures - longer mean free paths regime. In this range, Tonks and Langmuir² have shown that the density profile can be described approximately by the series

$$n = n_0 (1 + x + x^2 + \dots)$$

This is true only when the ion generation is proportional to the electron density.

(3) Very low pressures - long mean free paths regime. The plasma sheath equation for this use has been solved by J. V. Parker³, when the ion generation is proportional to the electron density. The profile distribution is quite complicated in this regime and can be expressed in terms of two quantities:

$$M = \frac{\int_0^{r_p} n(r) r^3 dr}{r_p^2 \int_0^{r_p} n(r) r dr}$$

$$\text{and} \quad \beta^2 = \frac{n_0 e^2}{\epsilon_0 k T} (r_p / s_w)^2$$

So far the experimental investigations have been confined only to the high pressure regime. They reveal that the distribution profile is approximately of the Schottky type at pressures above 1 mm.

Agreement is poorer as the pressure decreases. Investigations are now being made at lower pressures ranging from 50μ to $10^{-3}\mu$ of Hg.

A technical report on this problem is under preparation. A theoretical study of the collisionless plasma sheath equation for a spherical geometry is being attempted by Rama Rao.

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 - 2 L. Tonks and I. Langmuir, Physical Review, Vol. 34, 1929, p. 876.
 - 3 J. V. Parker, Physics of Fluids, Vol. 6, 1963, p. 1657.
 - 4 "Microwave Measurements of Electron Density Profiles in Plasmas," G. E. Stewart and Z. A. Kaprielian, Phenomenes d'ionisation dans les gaz, (Paris 1963), Vol. IV, July.
23. Theoretical Studies on an Antenna Immersed in a Plasma.
A. D. Wunsch, NASA Grants NSG-579, NGR-22-007-056 and NSG-685

Numerous attempts at ionospheric exploration are being made using satellite-borne antennas. Generally, the intention in such experiments is to determine the constitutive parameters (electron density, magnetic field strength, temperature, etc.) of the ionosphere through the use of impedance measurements on the antennas. The purpose of the present research is to establish a theory relating these

impedance measurements to the ionospheric parameters.

One aspect of the problem which has been completed, treats the ionosphere as a cold, homogeneous, anisotropic medium. The plasma is characterized by a dielectric tensor, and the antenna is regarded as a strip of electric current. The radiation resistance of the antenna has been determined with a current perpendicular to the magnetic field of the surrounding medium. A paper reporting the results of this investigation has been accepted for publication by the Canadian Journal of Physics and will appear shortly.

Another aspect of this research which is close to completion treats the plasma as a homogeneous, isotropic medium of finite temperature. In this investigation expressions have been obtained not only for the impedance of a cylindrical dipole antenna immersed in plasma but also for the distribution of current along the antenna. At present, the validity of using various simple, assumed current distributions as a means for obtaining values of the antenna impedance is being examined.

24. Coupled Monopoles in a Plasma. J. C. Robertson,

NASA Grant NGR-22-007-056.

Extensive theoretical and experimental work has been completed on the properties of parallel coupled monopole antennas in a dissipative dielectric medium. Plans are in progress to investigate experimentally and theoretically the related problem of two coupled monopoles immersed in a plasma. In particular, the distributions of current and charge along the antennas and their admittances will be investigated.

25. Transmission Lines in Plasmas. W. A. Saxton and Y. S. Yeh,
 NASA Grant NGR-22-007-056, Contract Nonr 1866(32) and
 U. S. Army Electronics Command, Fort Monmouth.

This study concerns the interaction between an open, balanced two-wire transmission line and a slightly ionized gaseous plasma. After a fundamental investigation of TEM transmission line waves through the plasma several possible applications will be considered, including use of a transmission line in plasma diagnostics and the feasibility of coupling plasma RF noise directly to such a line which could serve as a coupling element between the plasma and a radiating antenna in a penetrating aid device.

In the initial experimental set-up the transmission line passes through the positive column of a hot-cathode dc discharge-tube plasma in a direction transverse to the discharge. TEM waves propagating along the line experience a continuous variation of electron density. The effects of various density profiles on the transmission and reflection coefficients are being analyzed and solved rigorously by the integral equation method and through numerical computation.

If one assumes a Bessel-function radial electron density distribution with a plasma frequency $w_p = 900$ MHz at the center of the tube, corresponding to an electron density $n_o = 10^{10}/\text{c. c.}$, the expected phase shift at $w_p/w = 0.9$ would be 24° , where w is the electromagnetic frequency. On the other hand, the commonly used phase integral method would yield a shift of 19° . At $w_p/w = 1$, the two are given by 31° and 22° respectively. Such a typical discrepancy

comes mainly from the continuous multiple reflections in the plasma which are not negligible if the variation in density profile is not gradual compared to the RF wavelength.

Two baluns have been constructed to facilitate balanced measurements on a slotted coaxial line. The effects of the balun junction are determined by Deschamp's method so that they can be represented by a scattering matrix. Solution of the integral equation provides another scattering matrix for the plasma junction and completes the description of the experimental set-up.

Among the further steps in this investigation are the assumption of different plasma electron-density profiles and their effects on the transmission line parameters. Also planned is a comparison between experimental and theoretical scattering matrices and values of phase shift and attenuation through the discharge.

26. Radiation From A Circular Slot in a Warm Plasma.

Y. S. Yeh, Contract Nonr 1866(32).

The geometry of the problem under investigation consists of a circular slot in a ground plane which is covered by a dielectric layer and a homogeneous warm plasma region. The problem is of interest in that it takes into account both the plasma sheath effect and the finiteness of the source.

The formal solution is obtained using the hydrodynamic equations subject to proper boundary conditions. The field distribution can then be expressed by integral representations that are evaluated by the method of steepest descent. When the thickness of the dielectric layer and the transverse dimension of the slot shrink to

zero, the problem reduces to that of the radiation of an electric dipole above a ground plane. The result in this case agrees with that obtained by R. M. Langelier and F. V. Schultz.*

The radiation resistance, the power patterns of the E. M. mode and the acoustic mode, and the influence of the slot radius on the various modes of propagation are examined. A technical report is in preparation.

27. Brush-Cathode Discharges . W. A. Saxton, NASA Grant

NGR-22-007-056 and U. S. Army Electronics Command, Fort Monmouth .

Until recently it has been very difficult to produce electrically quiet laboratory plasmas without instabilities and striations. However, the initial work by Persson at the National Bureau of Standards on cold-cathode "brush" discharges¹ unveiled the possibility of generating well behaved and controllable gaseous plasmas for research work.

In their initial forms, brush cathodes consisted of large numbers of small diameter metallic needles, similar to sewing needles, mounted on flat circular disks. Such a configuration sealed in an inert gas discharge tube, and used in conjunction with a similar unit at a lower electrical potential, was found to generate a large uniform electron beam with a corresponding negative glow which had a longitudinal dimension one or two orders of magnitude larger than for a normal hot- or cold- cathode discharge. This negative glow was essentially field-free and recombination-dominated, making it a practically uniform plasma.

*R. M. Langelier and F. V. Schultz, "Radiation From a Vertical Dipole in a Warm Plasma - Part I and Part II. " IEEE Trans. on Antennas and Propagation. Vol. AP-14, No. 2, March 1966, pp.207-218.

¹K. B. Persson, "The Brush Cathode Plasma - A Well-Behaved Plasma, " NBS Report 8452, U.S. Dept. of Commerce, National Bureau of Standards, Boulder Laboratories, Boulder, Colorado, September 1964.

Although the brush-cathode has created a great deal of interest and study, there are many conflicting theories on the nature of its operation. In addition, not enough comparative experimental data are available to analyze the critical parameters in brush-cathode construction, nor to predict and evaluate the properties of the plasmas that it produces. This investigation is an attempt to shed some light on the brush-cathode phenomenon through an organized and detailed experimental study.

Three pairs of brush configurations have been built to date for comparison; a fourth pair is under construction. Each pair is circular cylindrical with a nominal diameter of 2 1/4 inches and lengths varying from 2-3 inches. Cathodes in one pair consist of over 1300 1-5/6" stainless-steel sewing needles brazed on a stainless-steel backplate. These cathodes will be compared to so-called "inverse" brush cathodes in which over 1300 0.027" holes will be dulled 1 5/16" deep into a stainless-steel cylinder. A third pair of cathodes is of the inverse-brush type made of monel alloy, but with much fewer holes than the preceding, and of larger diameters - 3 mm diameter holes uniformly spaced on 5 mm centers drilled 1 5/16" deep. The fourth pair consists of circular cylinders with smooth flat faces also made of monel.

Each pair of cathodes will be studied in turn using a glass test chamber attached to a vacuum system. The test chamber will provide for separations of one to six inches between the two cathodes in the discharge. Five Langmuir probes have been built for electron

density and temperature measurements in the discharges. Three probes will be radially moveable in addition to being regularly spaced in the discharge column so that the entire discharge can be probed. Volt-ampere characteristics of the discharge will also be measured for each discharge condition.

Several discharge parameters will be considered for each pair of cathodes, including gas pressure, discharge current and voltage, electron temperature and density, and collision frequency. In addition, electron density profiles will be determined, cathode spacing will be varied, and different types of gas will be used. By using an on-line real-time computer program developed for this experiment it should be possible to reduce the vast amount of Langmuir-probe data that will be obtained with the several degrees of freedom in the experiment. The results should be an exhaustive compilation of brush-discharge data.

28. Acoustic Effects in Plasmas . William A. Saxton, NASA Grant NGR-22-007-056 and U. S. Army Electronics Command, Fort Monmouth.

Fully calibrated moving armature and solid-dielectric transducers were used to detect and measure acoustic waves in the 200-2000 c/s range in an effort to correlate sound waves with so-called traveling striations which often occur in gas discharges. Although many data have been taken thus far in various laboratory plasmas, no conclusions are yet possible. Complicating the attempt to tie the electrical and acoustical effects together is the apparent presence of standing acoustic waves in the particular geometrics of the discharge tubes.

One interesting application for the acoustic waves which accompany the discharge striations is in modulating electromagnetic waves which impinge on a gaseous plasma column. Recent theoretical and experimental work indicates a significant scattering of electromagnetic waves propagating through a plasma which has a periodically-varying electron density transverse to the direction of propagation. Such an effect could be useful in varying the lobe structure of microwave antennas, for example, if the plasma properties were controllable.

Density variations which result from moving striations are usually time-variant and unstable. Another way to use the modulation technique is to propagate sound waves into a quiet plasma using an acoustic transducer of the type developed by Saxton¹. This perturbation method has been shown to produce definite electron density and collision-frequency variations in the positive column of a dc discharge at audio-frequencies² and does lend itself to predictable and reliable control of these plasma parameters both in amplitude and frequency. Construction of an experiment to investigate the EM-wave modulation effect has begun and a discharge tube has been completed in which it will be possible to study the scattering of electromagnetic waves by plasma acoustic waves created internally by striations, externally by transducer inputs, and by artificially stimulated ion acoustic waves.

¹ Saxton, W. A., "Transducers for Exciting and Detecting Acoustic Waves in Discharge-Tube Plasmas," Journal of the Acoustical Society of America, Vol. 38, No. 6, Dec. 1965.

² Ingard, U., "Acoustic Wave Generation and Amplification in a Plasma," Physical Review, Vol. 145, No. 1, May 1966.

A third phase of this work is directed toward an experimental study of acoustic wave amplification in gaseous plasmas. Ingard² at M.I.T. has laid the theoretical groundwork for such an investigation by showing that, under certain conditions, the electrons in a weakly ionized plasma heat the neutral-gas component and either generate or amplify sound waves. An experiment has been designed which should serve to explore the amplification possibility.

Since the magnitude of any acoustic amplification is increased by enlarging the ratio of electrons temperature to neutral-gas temperature³ it is planned to immerse part of the plasma in cryogenic surroundings. For this purpose a U-shaped discharge tube is planned with the bottom part of the "U" immersed in liquid helium. A brush cathode discharge will be used to assure freedom from any acoustic waves generated by the plasma itself. Sound waves will be forced into the cryogenic portion of the plasma using a moving armature transducer. After the sound passes through the cryogenic interaction region, where most of the amplification should take place, it will be detected and measured at the other end of the U-tube with a solid-dielectric microphone. All components in the discharge-tube itself, are fully bakeable and consistent with proper vacuum techniques.

Typical conditions in a helium discharge include a pressure of one Torr, a neutral gas temperature of 10^0 K in the cryogenically-cooled portion of the tube, an electron temperature equal to 1000^0 K, and an electron density of 5×10^{10} electrons/c.c. In a U-shaped $1\frac{1}{2}$ " diameter tube of the proper length and geometry, under these conditions it should

² See Footnote (2)

³ Saxton, W. A., "Excitation of Acoustic Waves in Plasmas," Radio Science, NBS Journal of Research, Vol. 69D, No. 4, April 1965.

be possible to achieve gains of at least 5-10 db. Discharges with higher electron temperatures, such as conventional hot- or cold-cathode types, should yield significantly higher gains.

29. Effect of Surface Roughness on the Characteristic Impedance and Propagation Constant of Coaxial Transmission Lines. A. E. Sanderson, Contract Nonr-1866(32).

Sections of coaxial transmission line with accurately measured lengths and diameter ratios are used as calculable standards of microwave impedance by the National Bureau of Standards and other laboratories concerned with primary standards. However, the calculation of impedance from mechanical dimensions presently requires the implicit assumption of perfectly smooth conducting surfaces.

Experimental evidence that this assumption is not usually justified has turned up in the manufacture of these standard lines¹. Surface roughness excursions on the order of the skin depth, which according to first-order theory should have negligible effect, in one case perturbed the characteristic impedance and velocity of propagation by more than 0.1%. This is of course intolerable in a primary standard.

An investigation has been started to find a theoretical explanation for this effect, and to make supporting measurements. A search of the literature has turned up two important references that confirm the existence of such an effect through mathematical analyses of the boundary

¹ T. E. Mac Kenzie and A. E. Sanderson, "Some Fundamental Design Principles for the Development of Precision Coaxial Standards and Components," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-14, pp. 29-39, January 1966.

conditions that apply to statistically rough surfaces^{2,3}. At present effort is being directed toward development of a similar theory applicable to propagation of the TEM mode in a waveguide, and toward setting up the experimental system necessary to check this theory.

² T. B. A. Senior, "Impedance Boundary Conditions for Statistically Rough Surfaces," Appl. Sci. Res., Vol. 8, Section B, pp. 437-462, 1960.

³ E. Feinberg, "On the Propagation of Radio Waves along an Imperfect Surface," Journal of Physics, USSR, Vol. VIII, no. 6, pp. 317-330, 1944.

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